The Design of Sliding Mode Controller with Perturbation Estimator Using Observer-Based Fuzzy Adaptive Network

Min-Kyu Park, Min-Cheol Lee, and Seok-Jo Go

Abstract: To improve control performance of a non-linear system, many other researches have used the sliding mode control algorithm. The sliding mode controller is known to be robust against nonlinear and unmodeled dynamic terms. However, this algorithm raises the inherent chattering caused by excessive switching inputs around the sliding surface. Therefore, in order to solve the chattering problem and improve control performance, this study has developed the sliding mode controller with a perturbation estimator using the observer-based fuzzy adaptive network. The perturbation estimator based on the fuzzy adaptive network generates the control input for compensating unmodeled dynamics terms and disturbance. And, the weighting parameters of the fuzzy adaptive network are updated on-line by adaptive law in order to force the estimation errors to converge to zero. Therefore, the combination of sliding mode control and fuzzy adaptive network gives rise to the robust and intelligent routine. For evaluating control performance of the proposed approach, tracking control simulation is carried out for the hydraulic motion simulator which is a 6-degree of freedom parallel manipulator.

Keywords: sliding mode control, perturbation estimator, observer-based fuzzy adaptive network, weighting parameter, hydraulic motion simulator, 6-degree of freedom parallel manipulator

I. Introduction

Sliding mode control is very attractive method for nonlinear systems [1]-[3]. It has been confirmed as an effectively robust control approach for nonlinear systems against parameters and load variations. However, some bounds on system uncertainties must be estimated in order to guarantee the stability of the closed-loop system, and its implementation in practice will cause a inherent chattering problem, which is undesirable in application. To overcome these demerits, many researches are carried out. Lee and Aoshima [4] proposed a sliding mode control algorithm with two dead zones for reducing the chattering. However, this algorithm could not completely reduce the inherent chattering which was caused by excessive switching inputs around the sliding surface. And, Choi and Kim [5] proposed a fuzzy sliding mode control algorithm which was designed to reduce the inherent chattering of the sliding mode control by using the fuzzy rules. However, the number of inference rules and membership functions of the fuzzy-sliding mode controller should be determined only through the trial and error method by an expert who had the knowledge of systems.

Fuzzy control is the most effective method using expert knowledge without the parameters and structure of the nonlinear systems [6]. However, it is difficult to design and analyze the adequate fuzzy rules. Therefore, many researches have been carried out to optimize parameters of the fuzzy system. The neuro-fuzzy system [7] such as ANFIS (Adaptive Network based Fuzzy Inference System) is representative method [8]. The neuro-fuzzy system is obtained by embedding the fuzzy inference system into the framework of artificial neural network.

So, this study has developed the sliding mode controller with perturbation estimator using observer-based FAN (Fuzzy Adaptive Network). This control algorithm is designed to solve the chattering problem of a sliding mode control and select the adequate fuzzy parameters. The perturbation estimator generates the control input for compensating unmodeled dynamic terms and disturbance using the observer-based FAN. The weighting parameters of the observer based FAN are updated on-line by adaptive law in order to force the estimation errors to converge to zero. Therefore, the combination of sliding mode control and FAN (Fuzzy Adaptive Network) gives rise to the robust and intelligent routine. For evaluating control performance of the proposed approach, tracking control simulation is carried out for the hydraulic motion simulator which is a 6-degree of freedom parallel manipulator [9].

II. Observer based fuzzy adaptive network

1. System modeling

The dynamic equation of a nonlinear system with n-degree of freedom can be written as follow:

Fig. 1. Structure of observer based FAN.
\[ x_i = f_i(x) + \varphi_i(x) + \sum_{j=1}^{n} [b_{ij}(x)u_j + \varphi_{ij}(x)u_j] + d_i(t) \]

where, \( x = [x_1, ..., x_n]^T \) is the state vector, and \( x = [x_1', x_2']^T \). The term \( f_i(x) \) and \( b_{ij}(x) \) correspond to the elements of system matrix and those of control gain matrix, respectively, and these terms are the known values. The \( \varphi_i(x) \) and \( \varphi_{ij}(x) \) are uncertainties of \( f_i(x) \) and \( b_{ij}(x) \), respectively. And, \( d_i(t) \) is the control input. Perturbation is defined as the combination of all the uncertainties of Eq. (1).

\[ \Delta_i(x, t) = \varphi_i(x) + \sum_{j=1}^{n} [\varphi_{ij}(x)u_j] + d_i(t) \]

2. Sliding state observer

The control task is to drive the state towards a desired state in spite of these perturbations. It is assumed that the perturbations are upper bounded by a known continuous function of the states. A sliding state observer for SISO (single input single output) system is a robust observer which estimates the state of a nonlinear system. The state space representation of a second order SISO system is as

\[ \dot{x}_1 = x_2 \\
\dot{x}_2 = f(x) + b(x)u + \varphi(x)u + d(t) \]

\[ y = x_1 \]

where, \( x = [x_1', x_2'] \) is the state vector. The observer task is to estimate the state \( x \) in despite of the uncertainties. The sliding state observer is presented by Eq. (4) [10].

\[ \dot{x}_1 = \dot{x}_2 - k_1 \text{sat}(\dot{x}_1) - \alpha_1 x_1 \]

\[ \dot{x}_2 = f(x) + b(x)u - k_2 \text{sgn}(\dot{x}_1) - \alpha_2 x_1 \]

where \( k_1, k_2, \alpha_1, \alpha_2 \) are positive number and \( \dot{x}_1 = \dot{x}_1' \) is the estimation position error. Throughout the text, \( \dot{\cdot} \) refers to estimation errors whereas \( \cdot' \) symbolizes the estimated quantity. Using Eqs. (3) and (4), the estimation error dynamics are Eq. (5).

\[ \dot{x}_1 = \dot{x}_2 - k_1 \text{sat}(\dot{x}_1) - \alpha_1 x_1 \]

\[ \dot{x}_2 = f(x) + b(x)u - k_2 \text{sgn}(\dot{x}_1) - \alpha_2 x_1 \]

where, \( \Delta_0 \) is defined in Eq. (2) and the each difference \( f = f(\dot{x}') - f(x) \) and \( b = b(\dot{x}') - b(x) \) is assumed to be part of the uncertainty \( \varphi \) and \( \varphi_i \) of Eq. (2) respectively.

3. Description of fuzzy adaptive network

The fuzzy system consists of some fuzzy rules and a fuzzy inference system [11][12]. The fuzzy inference system uses the fuzzy rules to perform a mapping from an input linguistic vector \( z = [z_1, z_2, ..., z_n] \in \mathbb{R}^n \) to an output linguistic variable \( y \in \mathbb{R} \). The ith fuzzy IF-THEN rule is written as Eq. (6).

\[ R(i) : \text{if } z_1 \text{ is } A_{1i} \text{ and } z_2 \text{ is } A_{2i} \text{ and } ... \text{ and } z_n \text{ is } A_{ni} \text{ then } y \text{ is } B_i \]

where \( A_{1i}', A_{2i}', A_{ni}', \text{ and } B_i \) are fuzzy sets.

Firstly, \( \varphi_i \) is calculated by using product inference of Eq. (7). Secondly, fuzzy basis value (\( \tilde{\varphi}_i \)) is obtained by using normalization process of Eq. (8). Finally, the output of fuzzy logic system can be obtained by using singleton defuzzifier of Eq. (9) [12].

\[ \varphi_i = \sum_{h=1}^{H} \beta_{ih} \] (7)

\[ \tilde{\varphi}_i = \frac{\varphi_i}{\sum_{h=1}^{H} \varphi_i} \] (8)

\[ y(\tilde{x}) = \sum_{i=1}^{H} \tilde{\varphi}_i = \tilde{y} = \tilde{\varphi}_i \] (9)

where, \( \tilde{\varphi}_i(\tilde{x}) \) is membership function value of the fuzzy variable \( (z_i) \), \( h \) is the number of the total fuzzy IF-THEN rules, \( \varphi_i = [\varphi_{i1}, \varphi_{i2}, ..., \varphi_{in}] \) is adjustable parameter vector, and \( \tilde{\varphi}_i = [\tilde{\varphi}_{i1}, \tilde{\varphi}_{i2}, ..., \tilde{\varphi}_{in}] \) is fuzzy basis vector. The fuzzy logic approximator based on neural networks can be established [8]. Fig. 1 shows the configuration of the FAN. This network has five layers. At layer 1, nodes represent the values of the membership function of total linguistic variables. Usually, bell-shaped membership function is widely used. At layer 2, every node in this layer multiplies the incoming signals and sends the product result. At layer 3, every node in this layer calculates the fuzzy basis using normalization process. The ith node calculates the ratio of the ith rule's firing strength to the sum of all rules' firing strengths. At layer 4, every node including weighting factor (adjustable parameter) multiplies the fuzzy basis. At layer 5, the single node in this layer computes the overall output as the summation of all incoming signals.

4. Perturbation estimator using observer based FAN

In this section, our task is to use the FAN to approximate the perturbation function (\( \Delta_0 \)) in Eq. (2), and develop an adaptive control law to adjust the parameters of the FAN for the purpose of forcing the estimation error and estimation velocity error to converge to zero.

Fig. 2 shows the perturbation estimator using the observer based FAN. Input variables are estimation position and velocity and output variable is the estimated perturbation. The perturbation function is presented by Eq. (10).

\[ \tilde{\Delta}_0(\tilde{x}) = \tilde{y} \tilde{\varphi}_i(\tilde{x}) \] (10)

The adaptive law for updating adjustable parameters is chosen as Eq. (11).

\[ \dot{\varphi}_i = \lambda(w_1 \dot{x}_1 + w_2 \dot{x}_2) \tilde{\varphi}_i(\tilde{x}) \] (11)

where \( \varphi_0 \) and \( \varphi_0 \) are weighting factors, and \( \lambda \) is learning constant.
where the sliding function is presented by Eq. (12).

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 - k_1 \text{sat}(\dot{x}_1) - \alpha_1 x_1, \\
\dot{x}_2 &= f(x) + b(x) u - k_2 \text{sgn}(\dot{x}_1) - \alpha_2 x_1 + \tilde{\Delta}_x
\end{align*}
\] (12)

Therefore, the new observer errors dynamics become Eq. (13).

\[
\begin{align*}
\dot{\hat{x}}_1 &= \dot{\hat{x}}_2 - k_1 \text{sat}(\dot{\hat{x}}_1) - \alpha_1 \hat{x}_1, \\
\dot{\hat{x}}_2 &= -k_2 \text{sgn}(\dot{\hat{x}}_1) - \alpha_2 \hat{x}_1 - \tilde{\Delta}_x
\end{align*}
\] (13)

If the estimated perturbation value converges to the true perturbation value, the new observer structure (Eq. 12, and 13) is a better than a general state observer (Eq. 4, and 5) because this observer with perturbation estimator improves estimation accuracy of the states. And, it provides an on-line perturbation estimation scheme by using the FAN.

### III. Sliding mode control with perturbation estimator

In this section, a sliding mode controller with the perturbation estimator is designed. The estimation sliding function is defined as Eq. (14).

\[
\begin{align*}
\tilde{s}_j &= \tilde{s}_j + c_j \tilde{e}_j
\end{align*}
\] (14)

where, \(c_j\) is the desired control bandwidth and always positive, \(\tilde{e}_j = x_{ij} - x_{ij0}\) is the estimation position error and \([x_{ij0} x_{ij0}]^T\) is the desired motion cue for the \(j\)th degree of freedom. The actual sliding function is presented by Eq. (15).

\[
\begin{align*}
s_j &= \tilde{s}_j + c_j e_j
\end{align*}
\] (15)

where \(e_j = x_{ij} - x_{ij0}\) is the actual position error.

The control input \((u_j)\) of Eq. (1) is selected by using time derivative of the Lyapunov function candidate be given by \(\tilde{s}_j \tilde{s}_j < 0\) to satisfy the boundary layer attraction condition.

A desired \(\tilde{s}_j\) is selected as Eq. (16) [13].

\[
\begin{align*}
\tilde{s}_j &= -K_j \text{sat}(\tilde{s}_j)
\end{align*}
\] (16)

where, \(K_j\) is positive constant, and \(\text{sat}(\tilde{s}_j)\) is defined as

\[
\begin{align*}
\text{sat}(\tilde{s}_j) = \begin{cases} 
\frac{\tilde{s}_j}{|\tilde{s}_j|} & \text{if } |\tilde{s}_j| \geq \xi_j \\
0 & \text{if } |\tilde{s}_j| < \xi_j
\end{cases}
\end{align*}
\]

\(\text{sat}(\tilde{s}_j)\) is effectively used for anti-chattering problem. \(\xi_j\) is small constant as the boundary layer of sliding mode.

Using the results of previous sections such as Eq. (10), (11), and (12), it is possible to compute control input as Eq. (17).

\[
\begin{align*}
u_j = \frac{1}{b_j}(\tilde{x})^T f(\tilde{x}) + K_j \text{sat}(\tilde{s}_j) \cdot (k_j / \xi_j) \tilde{x}_j + \sum_{j=1}^{n}\left[ c_j(\tilde{x}_{ij0} - \tilde{x}_{ij0})\right]
\end{align*}
\] (17)

To summarize, Fig. 3 shows the overall scheme of the sliding mode control with perturbation estimator using observer based FAN proposed in this study.

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**IV. Control simulation**

This section presents the simulation results of the proposed sliding mode control with perturbation estimator using the observer based FAN. To evaluate performance of the proposed approach, tracking control simulation is carried out for the hydraulic motion simulator which is a Stewart platform manipulator.

1. **Modeling of Stewart platform**

The dynamic equation of the Stewart platform considering all inertia effect is known to be very difficult to derive. Lebret derived the dynamic equation using the Lagrange method and virtual work principle [14]. This equation can be written as Eq. (18).

\[
M_r(q) \ddot{q} + C_r(q, \dot{q}) \dot{q} + G_r(q) = J(q)^T U_r
\] (18)

\(q = [x, y, z, \alpha, \beta, \lambda]\) is coordinates vector of the upper centroid; \(\alpha, \beta, \lambda\) are the rotational angles about the \(x, y, z\) axes. \(M_r(q) \in \mathbb{R}^{6 \times 6}\) is the inertia matrix, \(C_r(q, \dot{q}) \in \mathbb{R}^{6 \times 6}\) corresponds to the centrifugal and Coriolis forces matrix,
$G_r(q)\in \mathbb{R}^{6\times 1}$ is the gravity force vector, $J(q)\in \mathbb{R}^{6\times 6}$ is the Jacobian matrix, and $U_r(q)\in \mathbb{R}^{6\times 1}$ is cylinder force vector. After some algebraic operation (\(I = J(q)\)) and kinematic transformation, Eq. (18) can be expressed as Eq. (19).

\[
\dot{\theta} + C_r(q, \dot{q}) + G_r(q) = U_r \quad (19)
\]

Fig. 4. Coordinate system of Stewart platform.

where, \(\dot{\theta}, C_r, G_r\) are derived as follows:

\[
\dot{\theta} = J^T(q)M(q)J^{-1}(q) \quad (20)
\]

\[
C_r(q, \dot{q}) = J^T(q)M(q)\frac{\partial}{\partial q}J^{-1}(q) + J^T(q)C(q, \dot{q})J^{-1}(q) \quad (21)
\]

\[
G_r(q) = J^T(q)G(q) \quad (22)
\]

where, \(l = [l_1, l_2, l_3, l_4, l_5, l_6]\) is cylinder length vector.

The cylinder dynamic equation is high order nonlinear equation. Assuming nonlinear part acts as a disturbance to the model, simple linear dynamics is obtained such as Eq. (20).

\[
M_A \ddot{\theta} + C_A \dot{\theta} + U_r = K_{SV}U_A \quad (20)
\]

\[
M_A \quad (20)
\]

The payload of the Stewart platform is about 250kg. The sampling time interval for control is selected by 10msec.

2. Control simulation

A simple tracking control is performed to check the proposed control algorithm. The tuned sliding state observer parameters of Eq. (12) \(k_1, k_2, \omega_1\), and \(\omega_2\) are 0.12, 2.4, 3, and 5, respectively. And, the tuned control parameters of Eq. (17) \(c_r, K_r\) are \(\xi_2\) are 20, 40, and 0.002, respectively.

The structure of perturbation estimator using the observer based FAN is a 2-6-9-9-1 system as shown in Fig. 2. The input variables are estimation cylinder position and velocity and output variable is the estimated perturbation. Here, the observer based FAN has 6 membership functions which are the bell shapes as Eq. (23) and Eq. (24).

\[
\hat{x}_1 := 1 + \frac{1}{1 + \left( \frac{\hat{\dot{\theta}}}{0.05} \right)^2} \quad (23)
\]

\[
\hat{x}_2 := 1 + \frac{1}{1 + \left( \frac{\hat{\dot{\theta}}}{0.1} \right)^2} \quad (24)
\]

Input variables of the observer based FAN are a normalized estimation position and velocity. The range of values for these variables is from -1 to 1. And, the learning constant \((\lambda)\) of Eq. (11) in on-line learning paradigm is set as 0.95. And the tuned weighting parameters \((\omega_1, \omega_2)\) are 20 and 100, respectively.

\[
\lambda = 0.95 \quad (25)
\]

The payload of the Stewart platform is about 250kg. The sampling time interval for control is selected by 10msec.

Fig. 5. Reference position trajectories.

Fig. 6. Reference velocity trajectories.
The reference position and velocity trajectory are shown in Fig. 5 and 6. The results of control simulation using the conventional sliding mode control are shown in Fig. 7 and Fig. 8. The conventional sliding mode control law is presented by Eq. (25) [16].

\[ u_j = -K_j \text{sat}(s_j) - c_j \dot{s}_j + I_d \]  

(25)

Here, \( I_d \) is desired acceleration trajectories. The peak error is about 10mm and the chattering occurs. The simulation results of the proposed sliding control algorithm are shown in Fig. 9 - Fig. 14. Fig. 9 and Fig. 10 show position error and velocity error for each cylinder. Fig. 11 and Fig. 12 show estimation position errors and velocity errors, respectively. And, Fig. 13 and Fig. 14 show actual perturbation and estimation perturbation for each cylinder. Fig. 14 shows that the proposed perturbation estimator using observer based FAN is rapidly converged to actual perturbation. In first stage of Fig. 9 and Fig. 10, peak position and velocity errors occur, because estimation velocity error of Fig. 10 is so large for 0.2sec. However, estimation errors and velocity errors are much smaller, as the estimated perturbation values obtained by using the observer based FAN are close to actual perturbation values. And, a inherent chattering occurred in the conventional sliding mode controller is reduced by using the sliding mode controller with perturbation estimator. Therefore, the proposed controller is able to provide a superior performance over the conventional sliding mode controller.

VI. Conclusion

This paper proposed the perturbation estimator using the observer based FAN, the sliding state observer for a nonlinear system, and a new robust and intelligent control algorithm named the sliding mode control with perturbation estimator using observer based FAN. The new observer structure is better than other state observer because this observer with perturbation estimator improves estimation accuracy of the states, in spite of nonlinear system. And, it provides an on-line perturbation estimation scheme by using the FAN. The proposed control algorithm can reduce the inherent chattering as estimating the states and compensating a perturbation in accuracy. And, a simple tracking control simulation was carried out for evaluating the proposed controller, the sliding state observer, and the perturbation estimator. The simulation results show that the designed sliding mode control can provide reliable tracking performance.
References


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