Optimization of Transient Stability Control
Part-I: For Cases with Identical Unstable Modes

Yusheng Xue, Wei Li, and David John Hill

Abstract: Based on the stability margin provided by the EEAC, the unstable contingencies can be classified into sets according to their unstable modes. This two-part paper develops a globally optimal algorithm for transient stability control to coordinate preventive actions and emergency actions. In the first part, an algorithm is proposed for a set of contingencies having identical unstable modes. Instead of iterations between discrete emergency actions and continuous preventive actions, the algorithm straightforwardly searches for a globally optimal solution. The procedure includes assessing a set of insufficient emergency schemes identified by the EEAC; calculating the related preventive actions needed for stabilizing the system; and selecting the scheme with the minimum overall costs. Simulations on a Chinese power system highlight its excellent performance. The positive results obtained are explained by analogizing settlements for 0-1 knapsack problems using the multi-points greedy algorithm.

Keywords: Extended equal area criterion (EEAC), nonlinear mixed programming, optimization, power systems, transient stability control.

1. INTRODUCTION

Two kinds of transient stability controls (TSC), namely preventive control (PC) and emergency control (EC), have been applied widely in modern power systems. PC usually employs continuous actions, influences pre-contingency operation and affects the system dynamics during and after any contingency. It is the most straightforward way to compare the actual operating point with the stability domain in the power-injection space. For maintaining the load supply, the generation reduction of the unstable cluster should be compensated by the increased output of others. The actions of EC are usually discrete, contingency specific and free from influencing the normal operation. Its decision-making can be modeled as a nonlinear integer-programming problem. Despite being relatively cost effective, the cost of PC spans the entire action period whether any objective disturbances occur or not, making it uneconomical or even infeasible in some circumstances. In contrast, EC actions usually cost dearly, and require restoration time following the actual occurrence of objective disturbances. Obviously, the coordination between PC and EC would lead to significant cost saving.

There have been many efforts to develop optimal PC and optimal EC methods [1-7]. Some of them simply check the stability of the static-security-constrained optimal solution. Others are limited to small systems and very few contingencies; their TSC decision-making relies on trial-and-error and heuristic knowledge. Moreover, PC and EC systems have been considered separately in research as well as in practice. Scala et al. [8] gave a general formulation to both stability-constrained economic dispatch and optimal EC. However, they are still treated as two individual problems. This contrasts with other areas of power system control such as voltage control where optimal coordination techniques are emerging [9]. If the original operating point $X_0$ is potentially unstable, PC may be activated prior to the contingency occurrence in order to move the system to a target point $X_T$ that may be yet insecure for some contingencies. When an unstable contingency-i is detected, relevant $E_i$ is activated immediately. Xue [10] proposed a general optimization framework for coordinating PC and EC. Based on it, this paper develops an optimal algorithm, which searches for global optimization in a hybrid decision space. Part-I deals with the optimal TSC for a set of contingencies having the identical unstable mode (UM), and Part-II deals with the cases having different UMs.
2. THE FOUNDATIONS AND FRAMEWORK

2.1. Foundations for TSC optimization

Coordination between PC and EC is a nonlinear mixed programming with both integer and continuous variables. This is a very difficult problem even for contingencies with an identical UM. Effective optimization algorithms should be founded on the mechanisms of transient stability, decomposition principle, and the optimization and coordination of pure PC actions with pure EC actions.

2.2. Mechanisms of transient stability

An unstable system always separates into a cluster of critical machines (cluster-S) and a cluster of non-critical machines (cluster-A) [11]. The UM is a very important concept used to characterize system separation. An action applied on cluster-S affects the stability in the opposite way to the same action applied on cluster-A. When a certain action enhances the stability of a UM, it may further the instability of other UMs.

UM identification is the key for (1) decoupling all the contingencies into sets, each of which is characterized by a unique UM; (2) selecting the search direction; and (3) coordinating the conflicts among the PC actions. The last point will be reported in Part-II of this two-part paper.

A stability margin of a disturbed trajectory is essential for defining the objective function of optimal TSC. The sensitivity analysis of the margin is the basis for choosing the appropriate searching step. On all accounts, quantitative methods for stability analysis and UM identification are foundations of an optimal TSC.

The extended equal area criterion (EEAC) is just such an approach [12]. It integrates the full system model in the multi-machine space, and then divides the complete disturbed trajectory into complementary clusters in all possible ways, corresponding to swing modes. For each mode, the trajectories are aggregated into those of an equivalent two-machine system. The necessary and sufficient conditions of stability are rigorously preserved into the most unstable mode, namely UM. In particular, the margin of the UM, labeled as $\eta$, represents the stability degree of the original system.

The EEAC breaks through the limitations of direct methods, thereby becoming applicable to any detailed models, complex scenarios and multi-swing stability while maintaining a manageable computational burden. It has been applied and is serving widely at home and abroad for power system planning, analysis, on-line operation, as well as for PC and EC.

Being quantitative and very informative, the EEAC can identify UM and sub-critical modes, decide both searching direction and searching step for optimizing TSC. Besides, it is fast enough for on-line tracking of system changes.

2.3. The optimization of pure PC actions

Based on the UM identification and quantitative assessment of the EEAC, the stability domain concept in the power-injection space is applied to PC in Xue et al. [13]. The optimization of PC, namely moving the operating point into this domain by redistributing the power-injection with minimum additional operating cost, is a nonlinear continuous-programming problem. The smaller the excursion from the original operating condition is, the lower the additional operating cost. The relevant software has been implemented in EMS for engineering services in the Dongbei grid and Guangxi grid.

2.4. The optimization of pure EC actions

The optimization of pure EC actions is a non-convex nonlinear integer-programming problem. Cheng and Xue [14] proposed a very effective algorithm, whose effectiveness was validated by numerous simulations and engineering applications. The ratio of finding the genuine optimum is over 98% while sub-optimal solutions were obtained for the other 2%. The relevant software has been implemented in adaptive system protection schemes for engineering services in the Shandong grid and Guangdong grid.

2.5. The optimization of PC and EC

Due to the complex mutual-influences between PC and EC, the globally optimal TSC is even more difficult to achieve. Xue [10] proposed a general optimization framework for the coordination aspect, where the task is formulated as a nonlinear hybrid-programming problem with both integer and continuous variables and with many stability constraints. The objective function is the sum of the daily cost for PC and the possibility-weighted cost for EC.

Unstable contingencies are firstly classified into subsets according to their UMs. During the optimal procedure, their UMs should be checked consistently to identify any possible changes resulting from the actions. Since all contingencies in such a subset have the same critical machines, a certain TSC action has the equivalent qualitative effects on their stability.

The two-layer TSC optimization consists of a lower layer for local coordination within each subset and an upper layer for global coordination among all subsets.

3. OPTIMIZING TSC FOR CASES WITH IDENTICAL UMS

3.1. Models for optimizing TSC

The EC+PC optimization can be formulated as:

$$
\min \ c(x_T) = \min \ (c_{PC}(x_T) + c_{EC}(x_T, e))
$$

(1)
\[
= \min (c_{PC}(x_T) + \sum_{i=1}^{n} \alpha_i c_{EC}(x_T, e_i)),
\]

\[s.t. \quad g(x_T) = 0, \quad (2)\]
\[h(x_T, e) \geq 0, \quad (3)\]

where \( C \) (or \( C_{PC} \)) is the total TSC (or PC) cost for a chosen time interval of interest; \( C_{EC} \) (or \( C_{EC} \)) is the total cost for EC actions \( e \) (or the cost of \( e_i \) for contingency-i); \( \alpha_i \) is the expected number of times of contingency-i during the assessment time interval; and \( X_T \) is a target point resulting from PC actions. \( 2 \) represents load flow constraints etc. Inequality \( 3 \) represents various security constraints including transient stability. \( c_{EC}(x_T) \) is the solution of the following sub-problem:

\[
\min c_{EC}(e_i(x_T)), \quad (4)
\]

\[s.t. \quad f(e) \geq 0, \quad (5)\]
\[\lambda \geq 0, \quad (6)\]

Inequality \( 5 \) represents capacity constraints of the emergency actions: the quantity of every action cannot exceed its maximum value and the total value of exclusive actions cannot exceed the relevant maximum value, for example generator tripping and fast-valving are mutually exclusive for a generator. Inequality \( 6 \) ensures that the system security has a positive margin for every pre-assigned contingency. \( 5 \) and \( 6 \) may be only subsets of \( 3 \).

3.2. Pictorial explanation

Fig. 1 illustrates the minimization of the total cost of TSC \( (1) \). Suppose the total generation of the critical machines \( (P_s) \) is increased according to some particular pattern; then we can associate each \( P_s \) with a \( X_T \) and re-express the cost as the sum of \( C_{PC}(P_s) \) and \( C_{EC}(P_{s.o}) \). The abscissa is the total generation of the critical machines \( (P_s) \); \( P_{s.o} \) is the original value that is assumed to be the optimal solution without stability constraints; the larger the excursion from \( P_{s.o} \), the higher the additional operating cost (see curve \( C_{PC} \)); \( P_{s.lim} \) is the stability limit with no EC action). If \( P_{s.lim} > P_{s.o} \), the system is stable without TSC. If \( P_{s.lim} < P_{s.o} \), TSC actions are necessary to keep the system stable under the relevant contingency.

There may be three choices for the unstable case: (1) only relying on PC, i.e. reducing \( P_s \) to be less than \( P_{s.lim} \); (2) only relying on EC, i.e. increasing \( P_{s.lim} \) to be larger than \( P_{s.o} \) with EC actions; (3) applying PC+EC, i.e. reducing \( P_s \) to a target value \( P_x \) beforehand, and applying proper EC actions in the event of contingency.

The sawtooth-like curve \( C_{PC}+C_{EC} \) represents the cost of the hybrid TSC, along which the point with the minimum ordinate value \( (P_{opt} \) in Fig. 1) corresponds to the optimal TSC. The optimization is just a procedure of searching for \( P_{opt} \).

4. THE ALGORITHM FOR CASES WITH IDENTICAL UMS

4.1. A common decomposition scheme

Decomposition-iteration methods are widely utilized in the large-scale system fields \[15\]. Zhao et al. \[16\] model an optimization problem as

\[
\min f(Y, X) = f_0(Y) + \sum_{i=1}^{J} f_i(Y, x_i),
\]

\[s.t. \quad g_k(Y) \leq 0 \quad k = 1, 2 \cdots K, \quad (7)\]
\[h_{i, j}(Y, x_i) \leq 0 \quad \forall i = 1, 2 \cdots I; \quad j = 1, 2 \cdots J,\]

where \( f(Y, X) \) is the objective function with two kinds of variables, i.e. \( Y \) and \( X = [x_1, x_2 \cdots x_J]^T \). The task can be decomposed into a two-layer optimization model as shown in Fig. 2. The common optimization steps are: (1) selecting an initial \( Y_0 \); (2) for sub-model-i at the layer-1, optimizing \( x_i \) under the resultant \( Y \); (3) at the layer-2, optimizing \( Y \) under the resultant \( X \); (4) repeating steps 2 and 3 until convergence is achieved.

The TSC optimization model, \( (4-6) \), is of the form of \( (7) \). However, since a PC action has different influences

Fig. 1. Coordination between \( C_{PC} \) and \( C_{EC} \).

Fig. 2. A common two-layer decomposition.
on EC effects of various contingencies, it is hard to select the modifying magnitude of PC actions. All EC schemes should be evaluated again if any change in PC actions occurs, which makes the computation burden extremely high. It is preferable if the number of iterations can be reduced or even avoided.

4.2. The problem-based decomposition for TSC

If no TSC action is needed, the original operating condition $x_o$ might seem to be the optimal one. If TSC actions are necessary, pure PC actions can be optimized as mentioned in Section 2.3, or pure EC actions can be optimized for an individual contingency as mentioned in Section 2.4.

Up to now, however, it is still a big challenge to optimize PC and EC collectively.

Once the EC actions are fixed, they can be treated as the details of the simulation scenarios to calculate the generation limits and the minimum additional PC actions. It is possible to formulate the optimization of PC and EC as an iteration procedure between a pure EC scheme and a pure PC scheme. However, extremely heavy computational burden and convergence difficulties make this infeasible in practice.

If good candidate EC schemes can be filtered out based on the nature of the TSC problem, and if the actual optimal or sub-optimal solution is among them, the number of iterations can be largely reduced or even eliminated.

During searching for the optimal pure EC scheme, it may be realized that many pure EC schemes have already been assessed. Although some of them are insufficient for stabilizing the system, they might be quite economical or even optimal if proper PC actions are taken jointly to make the system stable. Therefore, not only the optimal pure EC schemes, but also the insufficient ones are valuable candidates. The optimal PC+EC scheme is likely to be one of them.

Even if the discrete control space is high-dimensional and the number of possible EC schemes is big for one contingency, the optimal EC scheme as well as the insufficient EC schemes and stability limits can be obtained very effectively.

After candidate EC schemes for each contingency are selected respectively, the candidate PC+EC schemes for the whole set of contingencies can be obtained. Then, $x_T$ and the optimal PC+EC scheme can be identified among them.

Only if the effectiveness-cost-ratio of the resultant PC, namely $\Delta \eta_P / c_P(x_T)$, is much larger than the weighted ratio of EC, $\Delta \eta_E / c_E(x_T)$, it is preferable to repeat the procedure with $x_T$ as the new $x_0$.

4.3. Description of the proposed algorithm

There are three key factors that have great impacts on the algorithm performance when developing a globally searching algorithm. They are: (1) the initial candidate solutions; (2) the neighborhood of a candidate solution; (3) the searching strategies.

4.3.1 The initial candidate solutions

For a certain point in the power-injection space, such as the result of economic dispatch, EC optimization is performed for every pre-assigned contingency respectively. A set of insufficient pure EC schemes $U_i$ is found for contingency-i, where $i = 1, 2, \cdots, n$ and n is the number of pre-assigned contingencies.

Then, these insufficient EC schemes in $U_i$ are ranked according to their effectiveness-cost-ratio, i.e. $\Delta \eta / \Delta c$, where $\Delta \eta$ (or $\Delta c$) is the change of the stability margin (or cost) introduced by the scheme. The top ranked $m$ schemes consist of a set of candidates, labeled as $\Lambda_i$ for contingency-i. The additional PC actions, which are necessary and adequate for stabilizing the system, are estimated with the EEAC.

There are at most $m \times n$ feasible TSC schemes, which ensure the system stability under any one of the pre-assigned contingencies.

4.3.2 The neighborhood of a candidate solution

The neighborhood of a candidate scheme is the insufficient EC schemes in $U_i$, whose additional PC action is closest (either more or less) to that of the candidate scheme.

4.3.3 The searching strategies

From a global viewpoint, the searching for the optimal solution is performed for every candidate EC scheme in order to avoid trapping in local minima.

In a local sense, only those neighborhood schemes in the set $U_i$ with decrement in the total cost are considered. This is to enhance the search efficiency. Furthermore, to prevent cycling in the procedure, all the previously visited solutions should be memorized.

5. SIMULATIONS

An actual Chinese power grid, which is modeled with 75 generators, 730 buses and 2 external DC equivalent buses, is tested hereafter. Some severe contingencies correspond with a UM consisting of 10 machines.

Accurately pricing out PC actions and EC actions for TSC is very important for engineering practice [17]. Generally, EC costs should include option fees for making the service available and exercise fees for executing the actions [18], which might change according to the market operation.

Focusing on the optimization algorithm itself, the
cost for each action is set here as a time-invariant. By changing the set of constants, different test cases can be arranged just based on a unique \( X_o \) in order to evaluate the proposed searching algorithm.

The evaluation is performed by comparing the total cost of the recommended scheme (labeled as \( C_{rec} \)) to the ideal one resulting from exhausted simulations (labeled as \( C_{ide} \)). The cost for the pure EC (or PC) scheme is labeled as \( C_{EC} \) (or \( C_{PC} \)).

5.1. For a single contingency

Table 1 indicates the \( C_{rec} \), \( C_{ide} \), \( C_{EC} \) and \( C_{PC} \) for 8 cases. All cost constants of the PC actions remain unchanged for these situations. However, the cost constant of every EC action changes each time. The \( C_{rec} \) equals to \( C_{ide} \) for all cases, except for case-6, where the proposed algorithm gives a sub-optimal solution.

In case-2, the pure EC scheme is the ideal one. In other cases, the coordination scheme is more economical than pure EC and pure PC schemes.

Table 2 compares the integration trial-times of different searching methods. The “Lucky trials” means that the number \( m \) varies with both cases and contingencies, and is the minimum value needed for capturing the related optimal solution. These trials require perfect apriority knowledge, thus they are unrealistic and can only act as a reference.

The “General trails” fixes \( m=5 \) for all contingencies. During the procedure, many insufficient EC schemes might also be tested. The sufficient scheme along with the insufficient ones is taken together as the candidate EC schemes.

In an ascending order of their costs, the exhaustive procedure assesses all the pure EC schemes in turn. The searching stops upon locating the first sufficient one that can stabilize the system under the contingency. For each of them, the additional PC actions, which are sufficient and economical for stabilizing the contingency, and the total cost are calculated. The most economical scheme is assigned as the optimal solution, and the integration trial-times are also provided in Table 2 as another reference.

In most cases, the integration trial-times of the general trails procedure are quite close to those of the lucky trials and much less than those of the exhaustive search procedure.

In case-6, the general trails procedure only achieves a sub-optimal solution; the lucky trials uses an \( m=6 \).

5.2. For multi-contingencies with the same UM

Table 3 shows the results of optimal TSC for two contingencies, where \( a,b,\cdots,f \) are 6 different EC actions. The proposed algorithm obtains the real optimal solutions for all the cases. Since these cases only differ in the cost of the individual countermeasure, the integration trial-times of the exhaustive search remain unchanged.

Fig. 3 shows the situation in case-1, which favors the proposed procedure. The lowest cost of EC changes with both contingencies and PC actions, and many local minima of EC cost are excluded successfully by the proposed selection. Moreover, the initial solutions are quite close to the optimal one, which increases

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**Table 1. Comparison of the resultant costs.**

<table>
<thead>
<tr>
<th>Case</th>
<th>( C_{opt} )</th>
<th>( C_{rec} )</th>
<th>( C_{EC} )</th>
<th>( C_{PC} )</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>229</td>
<td>229</td>
<td>250</td>
<td></td>
<td>273</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td>189</td>
<td>189</td>
<td>200</td>
<td></td>
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<tr>
<td>4</td>
<td>187</td>
<td>187</td>
<td>200</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>219</td>
<td>219</td>
<td>250</td>
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<td>128</td>
<td>140</td>
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<td></td>
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<tr>
<td>7</td>
<td>179</td>
<td>179</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>167</td>
<td>167</td>
<td>170</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. Comparison of the integration times.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Integration trial-times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed procedure</td>
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<tr>
<td>Trials</td>
<td>Lucky</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
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<td>4</td>
<td>57</td>
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<td>6</td>
<td>59</td>
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<tr>
<td>7</td>
<td>53</td>
</tr>
<tr>
<td>8</td>
<td>57</td>
</tr>
</tbody>
</table>

**Table 3. Two contingencies with identical UM.**

<table>
<thead>
<tr>
<th>Case</th>
<th>EC actions</th>
<th>Integ. times</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a b c d e f</td>
<td>a b c d e f</td>
</tr>
<tr>
<td>2</td>
<td>0 0 1 0 1 0</td>
<td>0 0 1 0 1 0</td>
</tr>
<tr>
<td>3</td>
<td>1 0 0 0 0 0 0</td>
<td>1 0 0 0 0 0</td>
</tr>
<tr>
<td>4</td>
<td>1 0 1 0 0 0</td>
<td>1 0 1 0 0 0</td>
</tr>
<tr>
<td>5</td>
<td>1 0 1 0 0 0</td>
<td>1 0 1 0 0 0</td>
</tr>
<tr>
<td>6</td>
<td>0 0 1 1 0 0</td>
<td>0 0 1 1 0 0</td>
</tr>
<tr>
<td>7</td>
<td>0 0 0 0 1 1</td>
<td>0 0 0 0 1 1</td>
</tr>
<tr>
<td>8</td>
<td>1 1 0 1 0 0</td>
<td>1 1 0 1 0 0</td>
</tr>
</tbody>
</table>

**Fig. 3. The optimal PC+EC for two contingencies.**
the searching efficiency. The searches for the m initial schemes and the share of the tried PC solutions effectively avoid trapping into local minima.

6. DISCUSSION

The outstanding performances of the optimal TSC algorithm originate from the EC optimization algorithm [14], which selects EC schemes with the highest effectiveness-cost-ratio. This is similar to the greedy algorithm solving 0-1 knapsack problems [15].

For 600 knapsack problems produced randomly, the performance statistics of greedy algorithms are shown in Table 4, where m is the total number of the initial values [19]. When $m=1$, the real optimal solutions are obtained for 239 cases; 583 cases have sub-optimal solutions with the excursion less than 10% while maximum excursion is less than 25%. The performances are greatly improved if $m$ increases.

Although the number of EC schemes ($2^{N_i}$-1, where $N_i$ is the total number of EC actions available for contingency i) is usually very large, the number of the tested schemes is small. In fact, a large number of simulations indicate that the optimal scheme is often included in initial schemes.

7. CONCLUSIONS

Based on the quantitative assessment method EEAC, this paper proposes an optimization algorithm to coordinate PC and EC for contingencies with equivalent UM. This is the basis for Part-II of the two-part paper to finally deal with contingencies having various UMs for real world TSC problems. A large number of simulations on a Chinese power grid fully demonstrate its excellent searching performances.

REFERENCES

[16] W. Zhao et al., “Decoupling research in structural optimum design based on decomposition


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