Design of Fuzzy IMM Algorithm based on Basis Sub-models and Time-varying Mode Transition Probabilities

Hyun-Sik Kim and Seung-Yong Chun

Abstract: In the real system application, the interacting multiple model (IMM) based algorithm requires less computing resources as well as a good performance with respect to the various target maneuverings. And it further requires an easy design procedure in terms of its structures and parameters. To solve these problems, a fuzzy interacting multiple model (FIMM) algorithm, which is based on the basis sub-models defined by considering the maneuvering property and the time-varying mode transition probabilities designed by using the mode probabilities as inputs of a fuzzy decision maker, is proposed. To verify the performance of the proposed algorithm, airborne target tracking is performed. Simulation results show that the FIMM algorithm solves all problems in the real system application of the IMM based algorithm.

Keywords: Basis sub-models, fuzzy interacting multiple model algorithm, maneuvering target tracking, time-varying mode transition probabilities.

1. INTRODUCTION

The Kalman filter, which is well known as a recursive estimator based on optimal filter theory, has been widely used in target tracking. However, in the case that a single filter is used for maneuvering target tracking, its performance becomes inferior. For this reason, many kinds of Kalman filters have been studied in order to solve this problem. Among them, the interacting multiple model (IMM) algorithm is recognized to have a good performance although it is a sub-optimal filter [1-4]. In the IMM algorithm, if the target maneuvering is similar to the output of a sub-model, the tracking error is small; otherwise, the error is big. For this reason, it requires many sub-model numbers in order to have a good performance with respect to the various target maneuverings. But it is not reasonable to use the algorithm with too many computing resources in the real system application.

To solve this problem, various IMM based algorithms have been suggested. Munir [5] proposed the algorithm that has sub-models determined by the estimated acceleration. Although it has a small number of sub-models as well as a good performance, it has difficulty in determining the acceleration levels according to the maneuvering property. Wang [6] proposed the algorithm that has sub-models whose process noise variances are adjusted by the fuzzy system. Although it has a small number of sub-models as well as a good performance, it has a difficulty in determining the fuzzy system according to the expert knowledge. As well, Lee [7] proposed the algorithm that has three optimal sub-models whose parameters are adjusted by the genetic algorithm (GA). Although it has a small number of sub-models as well as a good performance, it still has a computational burden in optimizing sub-models by the GA. However, the performance of the IMM algorithm depends on the mode transition probabilities as well as the sub-models, i.e., if the mode transition probabilities are adjusted, the performance of the IMM algorithm is better than that of the conventional IMM algorithm. The performance of the IMM algorithm depends on the mode transition probabilities as well as the sub-models, i.e., if the mode transition probabilities are adjusted, the performance of the IMM algorithm is better than that of the conventional IMM algorithm. Campo [8] proposed the algorithm that adjusts the mode transition probabilities by the sojourn time dependent Markov model switching. Although it has superior performance, it has difficulty in determining the design parameters.

To resolve these problems, a fuzzy interacting multiple model (FIMM) algorithm, which is based on the basis sub-models defined by considering the maneuvering property and the time-varying mode transition probabilities designed by using the mode probabilities as inputs of a fuzzy decision maker, is proposed.

The design procedure of the FIMM algorithm encompasses the following contents: the practical definition method of the basis sub-models defined by
considering the maneuvering property; and the
detailed design method of the time-varying mode
transition probabilities designed by using the mode
probabilities as inputs of a fuzzy decision maker.

The proposed algorithm has four major advantages:
1) it has less computing resources because the number
of the basis sub-models is determined by the
maneuvering property 2) it has good performance
because the mode transition probabilities are adjusted
by the fuzzy decision maker 3) it has a simple fuzzy
partition with a small number of parameters and a
simple fuzzy rule that has the small rule number
because the mode probabilities are normalized values
and the sum of them is 1.0, and 4) it easily extends the
simplified fuzzy reasoning method [9,10] because the
mode transition probabilities have the form of a
matrix.

The IMM algorithm is introduced in Section 2. The
design of the FIMM algorithm is described in Section
3, and the simulation results of the FIMM algorithm
with respect to the various target maneuverings are
presented in Section 4. Finally, the conclusions are
summarized in Section 5.

2. IMM ALGORITHM

In this section, the main elements of the IMM
algorithm, which is based on the Kalman filter, are
introduced.

The IMM algorithm is recognized to have a good
performance with respect to the various target
maneuverings although it is a sub-optimal filter based
on the Markov chain whose transition depends on the
latest state. The detail contents are well explained in
[3], and the main elements of the IMM algorithm are
as follows:

The mode transition probabilities, which are related
to the Markov chain, are defined as

\[ P_{ij} = \sum_{k=1}^{r} p_{ij} \mu_{i}(k-1). \]  \hspace{1cm} (3)

And then, the mixed state and the state covariance
are defined as

\[ \hat{x}^j(k|k) = \sum_{i=1}^{r} \hat{x}^i(k|k) \mu_j(k-1|k), \]  \hspace{1cm} (4)

\[ P^j_0(k|k) = \sum_{i=1}^{r} \mu_j(k-1|k) \left\{ P^i(k|k) \right\} \]  \hspace{1cm} (5)

\[ + \left[ \hat{x}^j(k|k) - \hat{x}_0^j(k|k) \right] \left[ \hat{x}^j(k|k) - \hat{x}_0^j(k|k) \right]^T \].

where \( \hat{x}^j(k|k) \) is the state vector at the scan \( k \).

Also, the mode probability is defined as

\[ \mu_j(k) = \frac{1}{c} \Lambda_j(k) c_j, \]  \hspace{1cm} (6)

where \( c \) is a normalization constant.

\[ c = \sum_{j=1}^{r} \Lambda_j(k) c_j, \]  \hspace{1cm} (7)

and \( \Lambda_j(k) \) is a likelihood function defined as

\[ \Lambda_j(k) = \frac{1}{\sqrt{(2\pi)^{n_z} S_{j}(k)}} \exp \left\{ -\frac{1}{2} v^T_j(k) S_{j}^{-1}(k) v_j(k) \right\}, \]  \hspace{1cm} (8)

\( v_j(k) = z(k) - \hat{z}_j(k-1|k) \), \( S_{j}(k) \) is the innovation
covariance that includes the measurement covariance,
and \( n_z \) is the dimension of measurement vector \( z(k) \).

Finally, the combined state and the state covariance
are defined as

\[ \hat{x}(k|k) = \sum_{j=1}^{r} \hat{x}^j(k|k) \mu_j(k), \]  \hspace{1cm} (9)

\[ P(k|k) = \sum_{j=1}^{r} \mu_j(k) \left\{ P^j(k|k) \right\} \]  \hspace{1cm} (10)

\[ + \left[ \hat{x}^j(k|k) - \hat{x}(k|k) \right] \left[ \hat{x}^j(k|k) - \hat{x}(k|k) \right]^T \].

From the above mentioned equations, we note that
the performance of the IMM algorithm depends on the
mode transition probabilities as well as the sub-
models, i.e., if the target maneuvering is similar to the
output of a sub-model, the tracking error is small;
otherwise, the error is relatively big; and if the values
of a column in (1) are increased, the corresponding
sub-model is strongly reflected in generating the
combined state in (9); if the values of all columns are equally assigned, all sub-models are equally reflected.

3. DESIGN OF FIMM ALGORITHM

In this section, a FIMM algorithm, which is based on the basis sub-models defined by considering the maneuvering property and the time-varying mode transition probabilities designed by using the mode probabilities as inputs of a fuzzy decision maker, is designed.

The one cycle FIMM algorithm that has the fuzzy decision maker is shown in Fig. 1.

The detail design procedure of the FIMM algorithm is divided into the following two phases:

In the first phase of the design procedure, the practical definition method of the basis sub-models, which is defined by considering the maneuvering property, is described as follows:

Generally, the maneuvering property can be expressed by

$$\text{Maneuvering Property } = f(v, a, \omega, T, \sigma_w),$$

where $v$ is the target speed, $a$ is the target acceleration, $\omega$ is the target angular velocity, $T$ is the sampling period, and $\sigma_w$ is the standard deviation of the measurement noise.

In addition, the kinematic models can be divided into four types: a constant velocity (CV) model, a singer (SG) model [4], a constant acceleration (CA) model, and a coordinated turn (CT) model.

However, if the maneuvering property and the kinematic models are considered in the definition of

sub-models by designer, the definition is executed by the method that is shown in Fig. 2.

This method explains that the maneuvering property is closely related with elements such as the target speed, target acceleration, target angular velocity, sampling period, and standard deviation of the measurement noise, i.e., if the sampling period is small or the measurement noise is large, the number of sub-models can be reduced because some unnecessary sub-models may exist.

According to the analysis of the above mentioned definition method, the kinematic models can be interpreted as the acceleration models that have different acceleration rates and axis-coupling rates as follows:

$$CV < SG \leq CA < CT.$$  \hspace{1cm} (12)

This relation implies that SG and CA models can be unnecessary sub-models because they can be made by

![Fig. 1. FIMM algorithm (one cycle).](image1)

![Fig. 2. Sub-model definition method.](image2)
the weighted sum of the CV and CT models that can be candidates for the basis sub-models. And the \( \omega \) of the CT model can be determined as the maximum tuning rate of the desired target, which is generally known although its tuning direction is not known.

Therefore, two basis sub-models composed of CV model and CA model can be sufficient for tracking of the vertical maneuvering target and three basis sub-models composed of one CV model and two CT models can be sufficient for tracking of the horizontal maneuvering target in the horizontal plane.

Consequently, the number of the basis sub-models is small. Note that it solves the problem of less computing resources in the real system application of the IMM based algorithm.

In the second phase of the design procedure, the detailed design method of the time-varying mode transition probabilities, which is designed by using the mode probability as inputs of a fuzzy decision maker, is described as follows:

To adjust the mode transition probabilities in (1), the performance index is needed for evaluating each sub-model. However, the mode probability plays a role in evaluating each sub-model because the mode probability in (6) includes the likelihood function in (8). Therefore, the mode probability is used as a fuzzy input.

The fuzzy partition that has five bell-shaped membership functions is shown in Fig. 3.

This partition has the membership function that is defined as

\[
f_j(\mu_j) = \exp\left(-\frac{(\mu_j - c_i)^2}{\sigma_i}\right),
\]

where \( \mu_j \) is the mode probability of \( j \)-th sub-model, \( c_i \) and \( \sigma_i \) are respectively the center and width of \( i \)-th membership function.

In addition, it employs the facts that the mode probabilities in (6) are normalized values and the sum of them is 1.0. This is directly related with using the mode probabilities as fuzzy inputs. From these facts, the centers of DB and RB membership functions are respectively set to 1.0 and 0.0, and the center of ZO membership function is easily set to \( c_{ZO} = 1/r \).

And the center of DM membership function is properly set by considering the similarity of sub-models, i.e., if the degree of similarity between sub-models is high, \( c_{DM} \) is close to \( c_{ZO} \); otherwise, it is far from \( c_{ZO} \). Then, the center of RM membership functions is set by the above mentioned facts:

\[
c_{RM} = (1-c_{DM})/(r-1).
\]

And the widths of the membership functions are equally set for the design simplicity:

\[
\sigma_i = \sigma.
\]

(16) should be properly set in order to let the partition have a meaning in terms of a continuous overlap of the membership functions.

Therefore, the fuzzy partition has both simple structure and two design parameters of \( c_{DM} \) and \( \sigma \).

The fuzzy reasoning method employs the simplified method whose consequent part has a constant value. Therefore, it easily extends a constant value to a matrix because the mode transition probabilities have the form of a matrix.

The fuzzy rule has the following form:

\[
R^n: \text{if } \mu_1 \text{ is } A_1 \text{ and } \mu_2 \text{ is } A_2 \text{ and } \ldots \text{ and } \mu_r \text{ is } A_r \text{ then } p_{ij} = p^n_{ij},
\]

where \( R^n(n = 0, 1, 2, \ldots, r) \) denotes the \( n \)-th fuzzy rule, \( A_j \) denotes the membership function of the \( j \)-th sub-model, and \( p^n_{ij} \) that comprises the fuzzy consequent part is expressed by

\[
p^n_{ij} = \begin{bmatrix}
c_1 & c_2 & \cdots & c_j \\
c_1 & c_2 & \cdots & c_j \\
\vdots & \vdots & \ddots & \vdots \\
c_1 & c_2 & \cdots & c_j
\end{bmatrix},
\]

where

\[
c_j = \begin{cases} 
1/r, & n = 0 \\
(1-c_{\text{max}})/(r-1), & n \neq 0 \text{ and } j \neq n \\
c_{\text{max}}, & n \neq 0 \text{ and } j = n
\end{cases}
\]

Therefore, the fuzzy reasoning method has both a simple structure and an one design parameter of \( c_{\text{max}} \).

The fuzzy rule for tracking of the maneuvering target in a horizontal plane has four rules that are expressed by

\[
R^0: \text{if } \mu_1 \text{ is } ZO \text{ and } \mu_2 \text{ is } ZO \text{ and } \mu_3 \text{ is } ZO \text{ then } p_{ij} = p^0_{ij},
\]
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\[ R^1: \text{if } \mu_1 \text{ is bigger than } DM \text{ and } \mu_2 \text{ is smaller than } RM \text{ and } \mu_3 \text{ is smaller than } RM \text{ then} \]
\[ P_{ij} = P_{ij}^1. \]

\[ R^2: \text{if } \mu_2 \text{ is bigger than } DM \text{ and } \mu_3 \text{ is smaller than } RM \text{ and } \mu_1 \text{ is smaller than } RM \text{ then} \]
\[ P_{ij} = P_{ij}^2. \]

\[ R^3: \text{if } \mu_3 \text{ is bigger than } DM \text{ and } \mu_1 \text{ is smaller than } RM \text{ and } \mu_2 \text{ is smaller than } RM \text{ then} \]
\[ P_{ij} = P_{ij}^3. \]

These rules include the following rules: if the dominant model exists, the values of the corresponding column in (1) are increased in order to strongly reflect the corresponding sub-model in generating the combined state in (9); otherwise, the values of all columns are equally assigned in order to uniformly reflect all sub-models. It enables that the adjusted mode transition probabilities are expressed by the form of the weighted sum of the consequent parts.

Therefore, the fuzzy rule has both simple structure and the rule number of \( r+1 \).

The fuzzy defuzzification is expressed by the following form:
\[ P_{ij}(k) = \frac{\sum_{n=1}^{l} w_n p_{ij}^n}{\sum_{n=1}^{r} w_n}, \quad (21) \]
where
\[ w_n = \prod_{i=1}^{r} m_{A_i}(\mu_i(k-1)). \quad (22) \]

This fuzzy decision maker replaces the time-invariant \( p_{ij} \) in (1)-(3) to the time-varying \( p_{ij}(k) \) in (21). It enables to let the FIMM algorithm have better performance than that of the conventional IMM algorithm.

Consequently, the fuzzy decision maker has simple structures and fewer design parameters as well as a good performance. Note that they solve the problems of both a good performance and an easy design procedure in the real system application of the IMM based algorithm.

From the above mentioned procedure, the FIMM algorithm, which is based on the basis sub-models defined by considering the maneuvering property and the time-varying mode transition probabilities designed by using the mode probabilities as inputs of a fuzzy decision maker, has been designed.

4. SIMULATION RESULTS

The performance of the FIMM algorithm is tested with the problem of tracking an airborne target that is moving, which is described by the constant velocity flight and the coordinated turn flight in the horizontal plane. This was also shown in [11,12].

The process equations, which are related to the constant velocity flight and the coordinated turn flight of the airborne target, are defined as
\[ x(k+1) = \begin{bmatrix} T & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} T^2/2 \\ T \\ T \end{bmatrix} v(k), \quad (23) \]
\[ x(k+1) = \begin{bmatrix} \sin \omega T / \omega & 0 & -(1-\cos \omega T) / \omega \\ \cos \omega T & 0 & -\sin \omega T \\ 0 & (1-\cos \omega T) / \omega & \sin \omega T / \omega \end{bmatrix} x(k) \]
\[ + \begin{bmatrix} T^2/2 \\ T \\ T^2/2 \end{bmatrix} v(k). \quad (24) \]

The state vector is defined as
\[ x(k) = \begin{bmatrix} \xi \\ \dot{\xi} \\ \eta \\ \dot{\eta} \end{bmatrix}, \quad (25) \]
where \( \xi \) and \( \dot{\xi} \) are respectively the position and velocity of the target with respect to the x-axis, and \( \eta, \dot{\eta} \) are respectively the position and velocity of the target with respect to the y-axis.

The measurement equation is defined as
\[ z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + w(k). \quad (26) \]

The simulation scenario of the maneuvering target is designed as follows:
A nonmaneuvering flight during scan 1 to 20 with a speed of 300 m/s; a 180° turning flight during scan 21 to 33 with a turning rate of 3.74°/s (2g acceleration); a nonmaneuvering flight during scan 34 to 53; a –180° turning flight during scan 54 to 66 with a turning rate of –3.74°/s; a nonmaneuvering flight during scan 67 to 86; a 180° turning flight during scan 87 to 112 with a turning rate of 1.87°/s; finally, a nonmaneuvering flight during scan 113 to 132.

In order to compare the proposed FIMM algorithm with conventional IMM algorithms, an IMM1 with no knowledge and an IMM2 with a heuristic knowledge
are considered. The initial state of the target in Cartesian coordinates is determined by

\[ x(0) = \begin{bmatrix} 30000 & -172 \\ 30000 & -246 \end{bmatrix}^T. \]

The process noise of the true system is zero and the true trajectory is shown in Fig. 4.

The sampling period of the sensor system is determined by \( T = 3.5 \).

The standard deviation of measurement noise in the sensor system is determined by \( \sigma_w = 30.0 \).

In order to track the target, the number of sub-models is \( r = 3 \), which is determined by

\[ M = \begin{bmatrix} \omega_1 = 0, & \omega_2 = 2g, & \omega_3 = -2g \end{bmatrix}^T. \]

The noise covariance is

\[
Q = \begin{bmatrix}
T^4/4 & T^3/2 & 0 & 0 \\
T^3/2 & T^2/2 & 0 & 0 \\
0 & 0 & T^4/4 & T^3/2 \\
0 & 0 & T^3/2 & T^2/2 \\
\end{bmatrix} \sigma_v^2, \quad (27)
\]

where \( \sigma_v = 0.004 \).

The standard deviation of measurement noise in the filter is the same as the sensor system.

The above parameters are equally applied to all algorithms. The other parameters are given in Table 1.

The performance of the FIMM is tested by 100 times Monte Carlo simulation. The results are shown in Figs. 5-9. These are the root of mean square error (RMSE), which is expressed by

\[
RMSE = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \left( x(k) - \hat{x}(k) \right)^T \left( x(k) - \hat{x}(k) \right)}. \quad (28)
\]

Figs. 5 and 6 respectively show the RMSE of target position and velocity with respect to \( \xi \)-axis. And Figs. 7 and 8 respectively show the RMSE of target position and velocity with respect to \( \eta \)-axis. While the performances of all algorithms have the total relation of \( \text{IMM1} << \text{FIMM} < \text{IMM2} \) in the case

<table>
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<th>Table 1. Parameters of algorithms.</th>
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<td>IMM1</td>
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<td>IMM’s ( P_{ij} ) =</td>
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<td>Fuzzy’s -</td>
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that a considerably dominant sub-model exists, they have the total relation of $IMM2 \ll IMM1 < FIMM$ in the case that an inconsiderably dominant sub-model exists during scan 87 to 112. These are caused by the facts that the FIMM has the time-varying probabilities while the IMM1 and IMM2 have the time-invariant probabilities. These mean that FIMM is most robust with respect to the overall target maneuvering because the basis sub-model in (12) and fuzzy decision maker in (21) act well.

Table 2 shows the numerical comparison of the FIMM and the conventional IMM algorithms. Each element is the average value of both the average value of RMSE in the case that an inconsiderably dominant sub-model exists and the average value of RMSE in the case that a considerably dominant sub-model exists. In terms of the performances, the position performances of the FIMM are better than the other algorithms. Especially, the velocity performances of the FIMM are superior to the other algorithms. And in terms of one cycle computing resources, the fuzzy decision maker of the FIMM requires the computing resource that is equivalent to that of one sub-model of the conventional IMM algorithm. Therefore, the FIMM algorithm requires the computing resources of $r+1$ sub-models while the other algorithms can require the computing resources of more than $2r$ sub-models in order to obtain the same performances in general.

These values quantitatively verify the fact that FIMM is most effective and robust with respect to the overall target maneuvering.

Fig. 7 shows the RMS value of the mode transition probabilities in (21). The FIMM algorithm is effectively adjusting the mode transition probabilities with respect to the change of the target maneuvering. In the case that a dominant sub-model exists, the values of the corresponding column in (1) are increased in order to strongly reflect the corresponding sub-model in generating the combined state in (9); otherwise, the values of all columns are equally assigned in order to equally reflect all sub-models.

Although the comparisons are not executed under the same conditions with respect to the IMM based algorithms because there does not yet exist the algorithm capable of solving all the problems in the real system application of the IMM based algorithm, they show well that the FIMM algorithm has meaningful terms.

5. CONCLUSIONS

In this paper, the fuzzy interacting multiple model
algorithm, which is based on the basis sub-models defined by considering the maneuvering property and the time-varying mode transition probabilities designed by using the mode probabilities as inputs of a fuzzy decision maker, has been proposed.

In the first phase of the design procedure, the practical definition method of the basis sub-models defined by considering the maneuvering property has been described in order to allow the algorithm to have fewer computing resources.

In the second phase of the design procedure, the detailed design method of the time-varying mode transition probabilities designed by using the mode probability as the input of the fuzzy decision maker has been described in order to let the algorithm have both a good performance and an easy design procedure.

The proposed algorithm has four major advantages: 1) it has less computing resources because the number of the basis sub-models is determined by the maneuvering property 2) it has a good performance because the mode transition probabilities are adjusted by the fuzzy decision maker 3) it has a simple fuzzy partition that has a small number of parameters and a simple fuzzy rule that has the small rule number because the mode probabilities are normalized values and the sum of them is 1.0, and 4) it easily extends the simplified fuzzy reasoning method because the mode transition probabilities have the form of a matrix.

To verify the performance of the proposed algorithm, airborne target tracking has been performed. The simulation results have shown that the FIMM algorithm solves all problems in the real system application of the IMM based algorithm. With the Monte Carlo simulation, the results have guaranteed the performances of the FIMM algorithm.

REFERENCES


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