**Abstract:** In this paper a new type of filter, called the $H_2/H_\infty$ FIR filter, is proposed for discrete-time state space signal models. The proposed filter requires linearity, unbiased property, FIR structure, and independence of the initial state information in addition to the performance criteria in both $H_2$ and $H_\infty$ sense. It is shown that $H_2$, $H_\infty$, and $H_2/H_\infty$ FIR filter design problems can be converted into convex programming problems via linear matrix inequalities (LMIs) with a linear equality constraint. Simulation studies illustrate that the proposed FIR filter is more robust against temporary uncertainties and has faster convergence than the conventional IIR filters.

**Keywords:** $H_2/H_\infty$ FIR filter, initial state independency, LMI, unbiased property.

1. INTRODUCTION

The estimation problem deals with recovering some unknown parameters or variables from measured information in physical or mathematical models. Among estimation problems, the state estimator, called the filter, has been widely investigated for wide applications. The performance of the filter is measured by stability, small error, and insensitivity or robustness to signal model uncertainties and disturbances.

For a small error, it is usual to require the filter to be unbiased. For stochastic systems, an unbiased filter means that no matter what the real state is, the filter will follow it on the average. This also means that if there is no noise in the systems the filter will follow the real state exactly. In a similar way to the stochastic case, filters for deterministic systems can adopt the unbiased property in a deterministic sense. The unbiasedness for deterministic systems requires the filters to match exactly the real states of systems with zero disturbances. In short, the unbiased property will be used even for deterministic systems throughout this paper. The terminology ‘deadbeat’ has also been used in other studies instead of ‘unbiased’ [1,2].

Some prefer finite impulse response (FIR) filters to infinite impulse response (IIR) filters for robustness and stability. FIR filters make use of a finite number of measurements and inputs on the most recent time interval $[k-N, k-1]$, called the receding horizon, or the moving window. FIR filters for signal reconstruction have long been researched. However, FIR filters for state reconstruction have recently been investigated [3-6]. It has been generally accepted that the FIR structure is more robust to temporary modeling uncertain parameters and numerical errors than the IIR structure. Additionally, bounded input bounded output (BIBO) stability is always guaranteed for FIR filters.

In conventional filters that estimate states, the initial state information is often assumed to be known, which in practice, is often not the case. Therefore, in this paper the initial state information is assumed to be completely unknown. That is, the suggested filters will be obtained independently of the initial state information.

In this paper a linear FIR filter that is independent of the initial state information is represented by

$$\hat{x}_k = \sum_{i=k-N}^{k-1} H_{k-i} y_i + \sum_{i=k-N}^{k-1} L_{k-i} u_i$$

at time $k$ for some gains $H_{k-i}$ and $L_{k-i}$. The filter gains $H_{k-i}$ and $L_{k-i}$ are independent of the initial state information.

Filter properties depend heavily on the performance criterion. In the $H_2$ performance criterion, the $H_2$ norm of the transfer function from the disturbance to the estimation error is minimized [7-9]. This approach has been widely used and researched because it is tractable mathematically. In the $H_\infty$ performance criterion, the worst case gain between disturbance and estimation error is minimized [10-14]. More recently,
there have been approaches that consider both of the performance criteria simultaneously [15]. In this paper, we will take those two performance criteria into account to obtain the optimal filter for state space models.

Existing FIR filters are mainly focused on the minimum variance criterion that is a special case of the $H_2$ performance criterion [3-6]. The $H_\infty$ FIR filtering problem was first considered in [16]. The $H_\infty$ FIR filter presented in [16] is obtained by repeatedly solving a finite horizon $H_\infty$ filtering problem. However, in practice it neither guarantees the $H_2$ norm bound nor has independence from the initial state. $H_\infty$ FIR filter for signal reconstruction was considered [17]. However, that result is not applicable to state reconstruction problems. To the best of our knowledge, existing FIR filters are mainly focused on the finite horizon $L_2$ performance criterion [3-6]. The phrase 'by design' means that the unbiased property of the proposed FIR filter is both unbiased and optimal by design for the given performance criterion. The phrase 'by design' means that the unbiased property and optimality are simultaneously built into the proposed FIR filter during its design.

The proposed $H_2$/$H_\infty$ FIR filter is both unbiased and optimal by design for the given performance criterion. The aim of this paper is to develop design methods for FIR filters with a batch form

$$\hat{x}_{k+1} = H\hat{y}_{k-1} + LU_{k-1}$$

as solutions to those three FIR filtering problems. $H$ and $L$ in (5) are the gain matrices of a linear filter represented by

$$H \triangleq [H_N, H_{N-1}, \ldots, H_1],$$

$$L \triangleq [L_N, L_{N-1}, \ldots, L_1].$$

$U_{k-1}$ and $Y_{k-1}$ are defined as

$$U_{k-1} \triangleq [u^T_{k-N}, u^T_{k-N+1}, \ldots, u^T_{k-1}]^T,$$

$$Y_{k-1} \triangleq [y^T_{k-N}, y^T_{k-N+1}, \ldots, y^T_{k-1}]^T.$$
length.

We require that the filter in (5) be independent of any a priori information about the horizon initial state, \( x_{k-N} \), by making a filter of FIR structure. Furthermore, we require an unbiased property that the FIR filter in (5) satisfies the following relation for the nominal system (3):

\[
\hat{x}_k = x_k \quad \text{for any } x_{k-N}.
\]

(8)

To determine the constraint required for (8) to be satisfied, denote the measurements on the most recent time interval \([k-N, k-1]\) in terms of the state \( x_k \) at the current time \( k \) as

\[
Y_{k-1} = \bar{C}_N x_k + \bar{B}_N U_{k-1} + (\bar{G}_N + \bar{D}_N) W_{k-1},
\]

where

\[
W_{k-1} = \begin{bmatrix} w_{k-N}^T & w_{k-N+1}^T & \cdots & w_{k-1}^T \end{bmatrix}^T.
\]

(9)

\[
\bar{C}_N, \quad \bar{B}_N, \quad \bar{G}_N, \quad \bar{D}_N \quad \text{are constant matrices obtained as follows:}
\]

\[
\bar{C}_N = \begin{bmatrix} CA^{-N} & \vdots & CA^{-2} & CA^{-1} \\ CA B & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ CA^{-1} & \vdots & \vdots & \vdots \end{bmatrix},
\]

(11)

\[
\bar{B}_N = \begin{bmatrix} CA B & \vdots & \vdots & \vdots \\ 0 & CA^{-1} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & CA^{-1} \end{bmatrix},
\]

(12)

\[
\bar{G}_N = \begin{bmatrix} CA B & \vdots & \vdots & \vdots \\ 0 & CA^{-1} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & CA^{-1} \end{bmatrix},
\]

(13)

\[
\bar{D}_N = \text{diag}(D, D, \cdots, D).
\]

(14)

For a nominal system (3) we obtain, from (9),

\[
\hat{s}_k = HY_{k-1} + LU_{k-1} = H\bar{C}_N x_k + H\bar{B}_N U_{k-1} + LU_{k-1}.
\]

Therefore, the constraints on \( H \) and \( L \) required to satisfy (8) are given by

\[
HC_N = I, \quad HB_N = -L.
\]

(15)

From (15), we rewrite the FIR filter in (5) as

\[
\hat{s}_k = H(Y_{k-1} - \bar{B}_N U_{k-1}), \quad H\mathcal{C}_N = I.
\]

(16)

The constraint \( H\mathcal{C}_N = I \) will be called the unbiased constraint in the sense that it is an unbiased constraint for the nominal system (3) with zero disturbance, but may not be an unbiased constraint for the system (2) with nonzero disturbance input.

Define \( T_H(z) \) as the transfer function from the disturbance input \( w \) to the estimation error \( e \) of an FIR filter (16). Then we can formulate three FIR filtering problems as follows:

- \( H_2 \) FIR filtering problem: Find the filter (16) that minimizes \( \| T_H(z) \|_2 \).
- \( H_\infty \) FIR filtering problem: Find the filter (16) that minimizes \( \| T_H(z) \|_\infty \) (or minimizes \( \| T_H(z) \|_2 \) subject to \( \| T_H(z) \|_\infty < \beta \)).

In the next section, we present the formulation of the above FIR filtering problems in terms of LMIs.

**Remark 1:** It is noted that \( A \) should be nonsingular to obtain \( \bar{C}_N, \bar{B}_N, \) and \( \bar{G}_N \). In case of high-order systems, the system matrix may be a sparse matrix and hence singular. It seems that the restriction of \( A \) being nonsingular is somewhat strong. However, if the system matrix is obtained from sampled-time systems, then it is represented as \( A = e^{A\Delta} \), where \( A \Delta \) is the system matrix for the continuous-time system and \( \Delta \) is the sampling period. In that case, \( A \) is always nonsingular.

### 3. \( H_2/H_\infty \) FIR Filtering Via LMIs

#### 3.1. Error dynamics of FIR filters

As a starting point we derive the transfer function \( T_H(z) \). The disturbance input \( w_k \) satisfies the following state model on \( W_{k-1} \)

\[
W_k = A_w W_{k-1} + B_u w_k,
\]

(17)

where

\[
A_w = \begin{bmatrix} 0 & I & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{pN \times pN}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I \end{bmatrix} \in \mathbb{R}^{pN \times p}.
\]

It follows from (9) that

\[
Y_{k-1} - \bar{B}_N U_{k-1} - \bar{C}_N x_k = (\bar{G}_N + \bar{D}_N) W_{k-1}.
\]

(18)

Pre-multiply (18) by \( H \). From (16), we obtain

\[
\end{equation}
\[ e_k = \hat{x}_k - x_k = H(\overline{G}_N + \overline{D}_N)W_{k-1}. \] (19)

From (17) and (19), \( T_H(z) \) is given by
\[ T_H(z) = H(\overline{G}_N + \overline{D}_N)(zI - A_N)^{-1}B_N. \] (20)

### 3.2. \( H_2 \) FIR Filtering

Given a system transfer function \( G(z) \triangleq \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = C(zI - A)^{-1}B, \)

it is well-known that \( G(z) \) is given by
\[ G(z) = P(zB^T) \]

where \( P \) is the controllability Grammian given by
\[ P = \sum_{i=0}^{\infty} A_i^T B B^T A^T. \]

and obtained as the solution to the following Lyapunov equation
\[ APA^T - P + BB^T = 0. \]

Therefore, we have the following theorem for the \( H_2 \) FIR filter:

**Theorem 1:** Assume that the following LMI problem is feasible:
\[
\begin{aligned}
\min_{F,W} & \quad tr(W) \\
\text{subject to} & \quad \begin{bmatrix} W & (FM + H_0)(\overline{G}_N + \overline{D}_N) \\ * & I \end{bmatrix} > 0,
\end{aligned}
\]

where \( H_0 = (\overline{C}_N^T \overline{C}_N)^{-1} \overline{C}_N^T M^T \) is the bases of the null space of \( \overline{C}_N^T \). Then the optimal gain matrix of the \( H_2 \) FIR filter of the form (20) is given by
\[ H = FM + H_0. \]

**Proof:** The constraint \( H\overline{C}_N = I \) is required for the FIR filter to be of the form (16). \( H_2 \) norm of the transfer function \( T_H(z) \) in (20) is obtained by
\[
\|T_H(z)\|^2 = tr(H(\overline{G}_N + \overline{D}_N)P(\overline{G}_N + \overline{D}_N)^T H^T),
\]

where \( P = \sum_{i=0}^{\infty} A_i' B_i B_i^T (A_i')^T \). Because \( A_i' = 0 \) for \( i \geq N \), we obtain
\[ P = \sum_{i=0}^{\infty} A_i' B_i B_i^T (A_i')^T = \sum_{i=0}^{N-1} A_i' B_i B_i^T (A_i')^T = I. \]

Therefore
\[
\|T_H(z)\|^2 = tr(H(\overline{G}_N + \overline{D}_N)(\overline{G}_N + \overline{D}_N)^T H^T). \]

Introduce a matrix variable \( W \) such that
\[ W > H(\overline{G}_N + \overline{D}_N)(\overline{G}_N + \overline{D}_N)^T H^T. \] (24)

Then \( tr(W) > \|T_H(z)\|^2 \). By the Schur complement, (24) is equivalent to
\[
\begin{bmatrix} W & H(\overline{G}_N + \overline{D}_N) \\ * & I \end{bmatrix} > 0.
\]

Therefore, by minimizing \( tr(W) \) subject to the equality constraint \( H\overline{C}_N = I \) and the above LMI, we obtain the optimal gain matrix \( H \) of the \( H_2 \) FIR filter. The equality constraint \( H\overline{C}_N = I \) can be eliminated by computing the null space of \( \overline{C}_N^T \). All solutions to the equality constraint \( H\overline{C}_N = I \) are parameterized by
\[ H = FM + H_0, \]

where \( F \) is a matrix containing the independent variables. Replacing \( H \) by \( FM + H_0 \), the LMI condition in (25) is changed into (22). This completes the proof.

**Remark 2:** \( \overline{C}_N^T \overline{C}_N \) should be nonsingular in order to obtain \( H_0 \). This, in turn, implies that \( \overline{C}_N \) should be full column rank. Assuming the nonsingularity of \( A \), \( \overline{C}_N \) is full column rank if and only if \( \overline{C}_N A^N \) is full column rank. \( \overline{C}_N A^N \) is represented as
\[ \overline{C}_N A^N = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix}. \]

If \( (A,C) \) is observable, \( \overline{C}_N A^N \) is full column rank if \( N \geq n \). This signifies that the horizon length \( N \) greater than \( n \) guarantees \( H_0 \) to exist.

**Remark 3:** Recalling that the square of the \( H_2 \) norm is the error variance due to white noise with unit intensity, we show that (23) holds as follows:
\[
\|T_H(z)\|^2 = \begin{bmatrix} e^T(k)e(k) \\ e^T(k)e^T(k) \end{bmatrix} = tr(e^T(k)e(k)) = tr(H(\overline{G}_N + \overline{D}_N)E\{W_{k-1}W_{k-1}^T\}(\overline{G}_N + \overline{D}_N)^T H^T) = tr(H(\overline{G}_N + \overline{D}_N)(\overline{G}_N + \overline{D}_N)^T H^T). \]
Define $\gamma^*_2$ to be the $\|P_H(z)\|_\infty^2$ due to the optimal $H_2$ FIR filter and define $\Xi_N$ as

$$\Xi_N \triangleq (\bar{G}_N + \bar{D}_N) \bar{G}_N \bar{D}_N^T.$$

The following theorem states that the optimal $H_2$ FIR filter can be obtained analytically.

**Theorem 2:** The optimal $H_2$ FIR filter gain is given analytically as

$$H = (\mathcal{C}_N^T \Xi_N^{-1} \mathcal{C}_N)^{-1} \mathcal{C}_N^T \Xi_N^{-1}$$

and therefore we have

$$\gamma^*_2 = \text{tr}(H \Xi_N H^T).$$  \hfill (27)

**Proof:** Construct a Lagrangian as

$$J = \text{tr}(H \Xi_N H^T) + \text{tr}(\Lambda (H \bar{C}_N - I)),$$

where $\Lambda \in \mathbb{R}^{n \times n}$ is a Lagrange multiplier. It is clear from (23) that $H$ minimizing the above Lagrangian is the gain matrix of the optimal $H_2$ FIR filter of the form (16). For optimality, we require that

$$\frac{\partial J}{\partial H} = 2H \Xi_N + \Lambda^T \bar{C}_N = 0, \quad (28)$$

$$\frac{\partial J}{\partial \Lambda} = (H \bar{C}_N - I)^T = 0. \quad (29)$$

From (28), we obtain

$$\bar{C}_N \Lambda = -2 \Xi_N H^T. \quad (30)$$

Pre-multiply $\bar{C}_N^T \Xi_N^{-1}$ to the left of both sides in (30).

Using $H \bar{C}_N = I$, we have

$$\bar{C}_N^T \Xi_N^{-1} \bar{C}_N \Lambda = -2 \bar{C}_N^T \Xi_N^{-1} \bar{C}_N H^T = -2 \bar{C}_N^T H^T = -2I.$$ 

Therefore $\Lambda = -2(\bar{C}_N^T \Xi_N^{-1} \bar{C}_N)^{-1}$ and

$$H = -\frac{1}{2} \Lambda^T \bar{C}_N^T \Xi_N^{-1} = (\bar{C}_N^T \Xi_N^{-1} \bar{C}_N)^{-1} \bar{C}_N^T \Xi_N^{-1}.$$ 

Substituting $H$ above into (23) yields relation (27). This completes the proof.

3.3. $H_\infty$ FIR Filtering

For the system transfer function

$$G(z) \triangleq \begin{bmatrix} A & B \\ C & D \end{bmatrix} = C(zI - A)^{-1}B + D$$

it is well known from the bounded real lemma that, given $\gamma > 0$, the following two conditions are equivalent:

1. $\|G(z)\|_\infty < \gamma.$
2. There exists an $X > 0$ such that

$$\begin{bmatrix} -X & XB & 0 \\ * & -X & 0 & C^T \\ * & * & -\gamma I & D^T \\ * & * & * & -\gamma I \end{bmatrix} < 0.$$ 

From this, we obtain the following theorem for the optimal $H_\infty$ FIR filter.

**Theorem 3:** Assume that the following LMI problem is feasible:

$$\min_{F,X,\gamma_\infty} \gamma_\infty \text{ subject to}$$

$$\begin{bmatrix} -X & XA & XB & 0 \\ * & -X & 0 & C^T \\ * & * & -\gamma I & D^T \\ * & * & * & -\gamma I \end{bmatrix} < 0,$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ and $M^T$ is the basis of the null space of $\bar{C}_N^T$. Then, the optimal gain matrix of the $H_\infty$ FIR filter of the form (16) is given by

$$H = FM + H_0.$$ 

**Proof:** From the bounded real lemma, the condition $\|P_H(z)\|_\infty < \gamma_\infty$ is equivalent to the condition under which there exists $X > 0$ such that

$$\begin{bmatrix} -X & XA & XB & 0 \\ * & -X & 0 & (\bar{G}_N + \bar{D}_N)^T (FM + H_0)^T \\ * & * & -\gamma_\infty I & 0 \\ * & * & * & -\gamma_\infty I \end{bmatrix} < 0.$$ 

The equality constraint $H \bar{C}_N = I$ can be eliminated in exactly the same way as in $H_2$ FIR filter. This completes the proof.

3.4. $H_2/H_\infty$ FIR Filtering

From the previous two subsections, the formulation of the $H_2/H_\infty$ FIR filtering problem via LMIs is obvious. Therefore, we obtain the following theorem for the $H_2/H_\infty$ FIR filter:

**Theorem 4:** Given $\alpha > 1$, assume that the following LMI problem is feasible:

$$\min_{W,X,F,\gamma_\infty} \gamma_\infty \text{ subject to}$$

$$\begin{bmatrix} W (FM + H_0)(\bar{G}_N + \bar{D}_N) \\ I \end{bmatrix} > 0,$$
\[
\begin{bmatrix}
-X & X A_c & X B_c & 0 \\
* & -X & 0 & (\bar{G}_N + \bar{D}_N)^T (FM + H_0)^T \\
* & * & -\gamma_{\infty} I & 0 \\
* & * & * & -\gamma_{\infty} I \\
\end{bmatrix} < 0,
\]

where \( H_0 = (C_N^T C_N)^{-1} C_N^T \) and \( M^T \) is the basis of the null space of \( C_N^T \). Then, the gain matrix of the \( H_2/H_\infty \) FIR filter of the form (16) is given by

\[
H = FM + H_0.
\]

**Proof:** The proof is obvious and is omitted.

The above \( H_2/H_\infty \) FIR filtering problem allows us to design the optimal FIR filter with respect to the \( H_\infty \) norm while assuring a prescribed performance level in the \( H_2 \) sense. By adjusting \( \alpha > 0 \), we can trade off the \( H_\infty \) performance against the \( H_2 \) performance.

### 4. NUMERICAL EXAMPLE

To illustrate the characteristics and validity of the proposed FIR filter, a numerical example is presented for a linear discrete-time invariant state space model taken from [3].

\[
x_{k+1} = \begin{bmatrix} 0.9950 & 0.0998 \\ -0.0998 & 0.9950 + \delta_k \end{bmatrix} x_k + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_k,
\]

\[
y_k = [1 \ 0] x_k + [0 \ 1] w_k,
\]

where \( \delta_k \) is an uncertain model parameter. Assuming \( \delta_k = 0 \), we obtained \( H_2 \) FIR filters, \( H_\infty \) FIR filters, and \( H_2/H_\infty \) FIR filters with \( \alpha = 1.05 \) for different horizon lengths \( N = 3, 4, \ldots, 10 \) using the results in Theorems 1, 3, and 4, respectively. Fig. 1 compares the \( H_2 \)-norms of \( H_2 \) FIR filters and \( H_2/H_\infty \) FIR filters with that of \( H_2 \) IIR filters. Similarly, Fig. 2 compares the \( H_\infty \)-norms of \( H_\infty \) FIR filters and \( H_2/H_\infty \) FIR filters with that of \( H_\infty \) IIR filters. Both figures show a similar trend: norms of FIR filters decrease as the horizon length increases. This means that the performances of FIR filters improve with the horizon length. It is naturally expected that the norm of FIR filters will approach the norm of IIR filters as the horizon length approaches infinity. The \( H_2 \)-norms of \( H_2/H_\infty \) FIR filters are always greater than that of \( H_2 \) FIR filters. Similarly, the \( H_\infty \)-norms of \( H_2/H_\infty \) FIR filters are always greater than that of \( H_\infty \) FIR filters. This is because \( H_2/H_\infty \) FIR filters are obtained by taking both performance criteria into account.

From these figures, we see that the performances of FIR filters can’t be better than those of IIR filters in
normal situations. However, it is noted that FIR filters can be a better choice than IIR filters in some special cases. One notable case is the situation where the system is subject to temporary parameter variation. As mentioned previously, FIR filters are known to be more robust than IIR filters against temporary modeling uncertainties because they utilize only finite measurements on the most recent horizon. To illustrate this feature, we have designed an $H_2/H_\infty$ FIR filter with $N = 10$, $\alpha = 1.05$, and $\delta_k = 0$ and applied it to a system that is subject to temporary parameter variation as follows:

$$
\delta_k = \begin{cases} 
-0.1, & \text{if } 100 \leq k \leq 150 \\
0, & \text{otherwise}.
\end{cases}
$$

Fig. 3 compares the estimation error in $x_1$ of the $H_2/H_\infty$ FIR filter with those of the $H_2$ and the $H_\infty$ filters of IIR structure where the disturbance input $w_k$ is given by

$$
w_k = 0.1 \begin{bmatrix} e^{-30k} \\ e^{-30k} \end{bmatrix}.
$$

Therefore $w_k$ is an exponentially decreasing signal. It is noted that the $H_\infty$ IIR filter demonstrates superior performance for the interval $0 \leq k \leq 100$. This is because $w_k$ is deterministic disturbance rather than stochastic noise. It is noted that the performance of the $H_2/H_\infty$ FIR filter for $0 \leq k \leq 10$ is notably inferior. This is because the FIR filter does not have enough data for normal operation. Usually, data corresponding to the horizon length are required for normal operation. For the time interval $100 \leq k \leq 150$, the system is subject to parameter variation and this in turn leads to temporary modeling uncertainty. The estimation error of the $H_2/H_\infty$ FIR filter is smaller than that of the IIR filters for this interval. For $k \geq 150$ where there is no modeling uncertainty and $w_k$ is very small, the convergence speed of $H_2/H_\infty$ FIR filter to the true state of the system is faster than IIR filters. This example clearly shows the relative merits of proposed FIR filters compared with IIR filters.

5. CONCLUSIONS

In this paper, a new type of filter called the $H_2/H_\infty$ FIR filter is proposed for discrete-time state space signal models. The filtering problem is formulated in terms of linear matrix inequalities. The proposed filter has many desirable properties, that is, the filter is linear with the most recent finite measurements and inputs, does not require a priori information of the horizon initial state, and has the unbiased property for zero disturbance. Furthermore, due to the FIR structure, the $H_2/H_\infty$ FIR filter is believed to be robust against temporary modeling uncertainties or numerical errors, while other IIR filters, such as Kalman filters and $H_\infty$ filters, may show poor robustness in these cases. The proposed $H_2/H_\infty$ FIR filter will be useful for many signal processing problems where signals are represented by state space models. In the current paper, we assume that a system matrix is nonsingular. Study on the FIR filters that does not require the nonsingularity of system matrices will be a challenging future work.

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